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ON LEFT BIDERIVATIONS IN SEMIPRIME SEMIRING

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ABSTRACT

Motivated by some results on Reverse, Jordan and Left Biderivations , in [5], the authors investigated a prime ring of characteristic $\neq 2,3$ that admits a nonzero Jordan left biderivation is commutative. Also, if R is a semiprime ring and B is a left biderivation, then B must be an ordinary biderivation that maps R into its center. In this paper, we also derived the same thing in semiprime semirings

Keywords: *Semiring, Derivation, Bi derivations, Left Biderivation, Jordan Left biderivation*.

I. INTRODUCTION

In this paper, S will be an associative semiring with center Z(S). we shall write the usual brackets [x,y] = xy – yx for all x,y in S, and in our proofs we will make extensive use of the basic commutator identities [x,yz] = [x,y]z + y[x,z] and [xz,y] = [x,y]z + x[z,y].

In this paper we work with certain kinds of biderivations. Biderivations were been studied in several papers b many authors. Let M be a subring of a ring R. Following M.Bresar, a biadditive map D: M X M → R is called a biderivation of S if it is a derivation in each argument. Recall that an additive map d : S → S is a derivation of a semiring S satisfies the Leibnitz rule d(ab) = d(a) b + a d(b), for all a,b ∈ S. This paper we will see that a prime semiring of Characteristic $\neq$ 2,3 that admits a non-zero Jordan left biderivation is commutative

II. PRELIMINARIES

Definition: 2.1

A **Semiring** (S,+,•) is a non-empty set S together with two binary operations, + and • such that
(1). (S,+) and (S,•) are a monoid with identity 0 and 1.
(2). For all a, b, c ϵ S, a . (b + c) = a . b + a . c and (b + c) . a = b . a + c . a

Definition: 2.2

A semiring S is said to be **2- torsion free** if 2x = 0 ⇒ x = 0, ∀x ∈ S.

Definition: 2.3

A semiring S is **Prime** if xSy = 0 ⇒ x = 0 or y = 0, ∀ x, y ∈ S and S is **Semi Prime** if x S x = 0 ⇒ x = 0, ∀ x ∈ S.

Definition: 2.4

An additive map d : S → S is called a **derivation** if d(xy) = d(x) y + x d(y), ∀ x, y ∈ S

Definition: 2.5

A mapping B: S X S → S is a **symmetric mapping** if B(x,y) = B(y,x), ∀ x, y ∈ S

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Definition: 2.6

A symmetric biadditive mapping D: S X S → S is called a **symmetric biderivation** if D(xy,z) = D(x,z) y + x D(y,z), ∀ x, y, z ∈ S. Obviously, in this case also the relation D(x,yz) = D(x,y) z + y D(x,z), ∀ x, y, z ∈ S

Definition: 2.7

A symmetric biadditive mapping B: S X S → S is called a **symmetric Jordan biderivation** if $B(x^2,y)$ = B(x,y) x + x B(x,y), ∀ x, y ∈ S. Obviously, in this case also the relation $B(x,y^2)$ = B(x,y) y + y B(x,y), ∀ x, y ∈ S

Definition: 2.8

A biadditive mapping B: S X S → S is called a **left biderivation** if B(xy,z) = x B(y,z) + y B(x,z) and B(x,yz) = y B(x,z) + z B(x,y), ∀ x, y, z ∈ S

Definition: 2.9

An additive mapping B: S X S → S is called a **Jordan left biderivation** if $B(x^2,y)$ = 2x B(x,y) and $B(x,y^2)$ = 2y B(x,y) , ∀ x, y, z ∈ S

III. ON LEFT BIDERIVATION

In this part, we will see that a prime semiring of Characteristic ≠ 2,3 that admits a non-zero Jordan left biderivation is commutative. Also if S is a semiprime semiring and B is a left biderivation, then B must be an ordinary biderivation that maps S into its center.

Lemma: 3.1

Let S be a Semiring, char (S) ≠2. If B: S X S → S is a Jordan left biderivation, then for all a,b,c,x ∈ S

i) B(ab+ ba, x) = 2a B(b,x) + 2b B(a,x)
ii) B(aba,x) = a^2 B(b,x) + 3ab B(a,x) – ba B(a,x)
iii) B(abc + cba,x) = (ac + ca) B(b,x) + 3ab B(c,x) + 3cb B(a,x) – ba B(c,x) – bc B(a,x)
iv) (ab – ba) a B(a,x) = a (ab-ba) B(a,x)
v) (ab – ba) (B(ab,x) – a B(b,x) – b B(a,x)) = 0

Proof:

i) Since B is a Jordan left biderivation, $B(a^2, x)$ = 2a B(a,x), ∀ x, a ∈ S

Replacing a by a+b, ∀ a, b, x ∈ S

$B((a+b)^2, x)$ = 2 (a + b) B(a+b, x)
= 2a $\{B(a+b,x)\}$ + 2b $\{B(a+b,x)\}$
= 2a B(a,x) + 2a B(b,x) + 2b B(a,x) + 2b B(b,a) ---------- (1)

Another way,

$B((a+b)^2, x)$ = B $(a^2 + b^2$ + (ab + ba), x)
= $B(a^2,x)$ + $B(b^2,x)$ + B(ab + ba, x) --------- (2)

Comparing (1) & (2)

$B(a^2,x)$ + $B(b^2,x)$ + B(ab + ba, x) = 2a B(a,x) + 2a B(b,x) + 2b B(a,x) + 2b B(b,x)

Since B is Jordan left biderivation,

2a B(a,x) + 2b B(b,x) + B(ab + ba, x) = 2a B(a,x) + 2a B(b,x) + 2b B(a,x) + 2b B(b,x)

⇒ B(ab + ba, x) = 2a B(b,x) + 2b B(a,x)

ii) From(i), For all a,b, x ∈ S

B(a(ab+ba) + (ab + ba)a,x) = 2a B(ab + ba ,x) + 2 (ab +ba) B(a,x)
= 2a(2a B(b,x) + 2b B(a,x)) + 2ab B(a,x) + 2ba B(a,x)
= $4a^2$ B(b,x) + 6ab B(a,x) + 2ba B(a,x) --------- (3)

On the other hand, B(a(ab+ba) + (ab + ba)a,x) = B($a^2b + aba + aba + ba^2, x$)
= B($a^2b + 2aba + ba^2, x$)= B($a^2b + ba^2, x$) + 2 B(aba,x)
= 2a^2 B(b,x) + 2 b B(a^2,x) + 2 B(aba,x) [∵ by (i)
= 2a^2 B(b,x) + 4ba B(a,x) + 2 B(aba,x) --------- (4)
Comparing (3) and (4) we get, 2B(aba,x) = 2[a^2B(b,x) + 3ab B(a,x) – ba B(a,x)]
Since char(S) $\neq$ 2, B(aba,x) = a^2B(b,x) + 3ab B(a,x) – ba B(a,x)

iii) Linearizing (ii) on a we get,

B((a+c)b(a+c),x) = $(a+c)^2$ B(b,x) + 3(a+c) b B(a+c,x) – b (a+c) B(a+c,x)
$\Rightarrow$ B(aba,x) + B(abc,x) + B(cba,x) + B(cbc,x) = a^2 B(b,x) + c^2B(b,x)+ 2acB(b,x) + 3ab B(a,x) + 3abB(c,x) + 3cbB(a,x) + 3cbB(c,x) – ba B(a,x) – ba B(c,x) – bc B(a,x) – bc B(c,x)
Using (ii) we get, B(abc + cba,x) = (ac+ ca) B(b,x) + 3ab B(c,x) + 3cb B(a,x) – ba B(c,x) – bc B(a,x)

iv) Let A = B(ab(ab) + (ab)ba,x)

Apply (iii) in A we get,
A = (a(ab) + (ab)a) B(b,x) + 3ab B(ab,x) + 3 (ab) b B(a,x) – ba B(ab,x) - b^2a B(a,x) ---------(5)
On the other hand, A = B($(ab)^2$ + a b^2a,x) = B($(ab)^2$,x) +B(a b^2a, x)
= 2ab B(ab,x) + a^2B(b^2,x) + 3ab^2B(a,x) - b^2a B(a,x)
= 2ab B(ab,x) +2 a^2bB(b,x) + 3ab^2B(a,x) - b^2a B(a,x) ---------(6)
Comparing (5) & (6),
abB(ab,x) –ba B(ab,x) = a^2bB(b,x) – aba B(b,x) – b^2aB(a,x) + bab B(a,x)
$\Rightarrow$ (ab-ba) B(ab,x) = a(ab-ba) B(b,x) + b (ab – ba) B(a,x) ---------(7)
Now add (ab – ba) B(a^2,x) on both sides we get,
(ab-ba) B(ab,x) + (ab – ba) B(a^2,x) = (ab – ba) B(a^2,x) + a(ab-ba) B(b,x) + b (ab – ba) B(a,x)
a(ab- ba)B(b,x) + b(ab- ba)B(a,x) +2a(ab-ba) B(a,x) = a(ab-ba)B(a,x) + a(ab-ba)B(a,x) + a(ab-ba)B(b,x) + b(ab-ba) B(a,x)
$\Rightarrow$ 2a(ab-ba) B(a,x) = 2a(ab- ba) B(a,x)
Since char(S) $\neq$ 2, a(ab-ba) B(a,x) = a(ab- ba) B(a,x)
Replace a by (a+b) in (iv), ((a+b)b-b(a+b)) (a+b)B(a+b,x) = (a+b) ((a+b)b – b(a+b)) B(a+b,x)
$\Rightarrow$ (ab- ba) (a+b) B(a+b,x) = (a+b) (ab – ba) B(a+b,x)
$\Rightarrow$ (ab- ba) a B(b,x) + (ab – ba) b B(a,x) = (ab – ba) B(ab,x)
$\Rightarrow$ (ab – ba) [B(ab,x) – a B(b,x) –b B(a,x)] = 0

Theorem: 3.2
Let S be a prime semiring, char (S) $\neq$ 2,3. Then if S admits a non-zero Jordan left biderivation B:S X S $\rightarrow$ S, S is commutative

Proof:
Step: I
If B(a,z) $\neq$ 0 for some a,z $\in$ S, then (a(ax-xa) – (ax-xa)a) = 0, for all x $\in$ S
For, let a $\in$ S be a fixed element and f: S $\rightarrow$ S be a mapping defined by f(x) = ax – xa, $\forall x \in S$
We know that,(ab – ba) a B(a,x) = a (ab – ba) B(a,x) ----------- (*)
Now (iv) of 3.1, can be written in the form f^2(x) B(a,z) = 0 ---------- (8)
Since f(x) is a derivation, f(xy) = f(x) y + x f(y)
Again using the derivation f^2(xy) = f (f(x) y + f(x f(y)) = f^2(x) y + f(x) f(y) + xf^2(y)
= f^2(x)y + 2 f(x) f(y) + x f^2(y)
From (8), f^2(xy) B(a,z) = 0
$\Rightarrow$ (f^2(x) y + 2 f(x) f(y) + x f^2(y)) B(a,z) = 0
$\Rightarrow$ (f^2(x)y + 2 f(x f(y)) B(a,z) = 0 [∵ f^2(y) B(a,z) = 0 by (8)] ---------- (A)
Replace y by f(yu), $\Rightarrow$ (f^2(x) f(yu) + 2 f(x) f(f(yu)) B(a,z) = 0
$\Rightarrow$ (f^2(x) [f(y) u + y f(u)] + 2 f(x) [f^2(yu)]) B(a,z) = 0
$\Rightarrow$ { f^2(x) f(y) u + f^2(x) y f(u) + 2 f(x) [f^2(y) u + 2f(y) f(u)] }B(a,z) = 0
$\Rightarrow$ [f^2(x) f(y) u + f^2(x) y f(u)] B(a,z) = 0 [∵ by (A)] ----------- (9)

Substituting f(u) for u in (9), we get [f^2(x) f(y) f(u) + f^2(x) y f(f(u))] B(a,z) = 0

$\Rightarrow$ (f^2(x) f(y) f(u)) B(a,z) = 0, again we replace y by f(y) in (9) we get

$\Rightarrow$ [f^2(x) f^2(y) u + f^2(x) f(y) f(u)] B(a,z) = 0

$\Rightarrow$ [f^2(x) f^2(y) u] B(a,z) = 0 , $\forall x, y, u \in S$ ----------- (10)

Since S is prime semiring and B(a,z) $\neq$ 0, then f^2(x) f^2(y) = 0, $\forall x, y \in S$

In particular $(f^2(x))^2$ = 0 as required.

Step: II

If a^2 = 0, then B(a,z) = 0, $\forall z \in S$

Let W = B(a (xay + yax) a,z)

Using (ii) in lemma 3.1, W = a^2B(xay + yax,z) + 3a (xay + yax) B(a,z) – (xay + yax) a B(a,z) ----------- (11)

Since a^2=0, B(a^2,z) = 0 $\Rightarrow$ 2 a B(a,z) = 0.

Since S is 2-torsion free $\Rightarrow$ a B(a,z) = 0 Using this in (11) we get,

W = 3a(xay + yax) B(a,z) = 3axay B(a,z) + 3ayax B(a,z) ----------- (12)

From (ii) in lemma 3.1, B(aya,z) = a^2B(y,z) + 3ay B(a,z) – ya B(a,z)

= 3ay B(a,z) [$\because$ a^2=0 & aB(a,z) = 0]

Using (iii) in lemma 3.1,

B(ax(aya) + (aya)xa,z) = (a(aya) + (aya)a)B(x,z) + 3(ax)B(aya,z) + 3ayaxB(a,z) - xaB(aya,z)

W = (a^2ya + aya^2) B(x,z) + 3ax (3ay B(a,z)) + 3aya xB(a,z) – xa(3ay B(a,z))

Since a^2=0, W = 9axay B(a,z) + 3ayax B(a,z) ---------- (13)

Comparing (12) & (13) we get, 3axay B(a,z) + 3ayax B(a,z) = 9axay B(a,z) + 3ayax B(a,z)

$\Rightarrow$ 6axay B(a,z) = 0

Since char (S) $\neq$ 2,3, axay B(a,z) = 0, $\forall x, y, z \in S$. Therefore either a=0 (or) B(a,z) =0

In any case B(a,z) = 0, $\forall z \in S$

Step: III

S is commutative.

Let $a, z \in S$ such that B(a,z) $\neq$ 0. From steps I & II we get,

B(a(ax-xa) – (ax-xa)a,z) = 0, $\forall x \in S$ ----------- (14)

B(a^2x – axa – axa + xa^2,z) = 0

B(a^2x + xa^2,z) – 2 B(axa,z) = 0

2a^2B(x,z) + 2x B(a^2,z) – 2[a^2B(x,z) + 3ax B(a,z)- xa B(a,z)] = 0

4xaB(a,z) – 6ax B(a,z) + 2xa B(a,z) = 0

6xa B(a,z) – 6ax B(a,z) = 6(xa-ax) B(a,z) = 0

(xa-ax) B(a,z) = 0 for all x in S

Now Replace x by yx, for all x,y in S, ((yx)a – a(yx)) B(a,z) = 0

Add and subtract the element yax B(a,x) in the last equation, we get

((yx) a – a(yx)) B(a,z)+ yax B(a,x) – yax B(a,x) = 0

y(xa-ax) B(a,z) + (ya- ay)xB(a,z) = 0

(ya-ay)xB(a,z) = 0. Since B(a,z) $\neq$ 0, it follows that a $\in$ Z(S).

Thus we have proved that S is theunion of its subsets Z(S) and Ker (B)={a $\in$ S/B(a,z)=0, $\forall z \in S$}

Hence S =Z(S) or Ker (B) = S. But our assumption B $\neq$ 0, then S = Z(S).

ie, S is commutative,

Theorem: 3.3

Let S be a semiring and B: S X S $\rightarrow$ S is a left biderivation.

(i).Suppose that S is a prime and B $\neq$ 0, then S is commutative

(ii).If S is semiprime semiring, then B is a biderivation that maps S into its center.

Proof:

Consider B(aba,x), for all a,b,x in S

Since B is a left biderivation, then B(a(ba),x) = aB(ba,x) + baB(a,x)

= ab B(a,x) + a^2B(b,x) + ba B(a,x) ----------- (15)

On the other hand, B((ab)a,x) = ab B(a,x) + a B(ab,x)

= ab B(a,x) + a^2B(b,x) + ab B(a,x) ----------- (16)

Comparing (15) & (16), we get

abB(a,x) + a^2B(b,x) + ba B(a,x) = ab B(a,x) + a^2B(b,x) + ab B(a,x)

$\Rightarrow$ (ab-ba) B(a,x) = 0, $\forall$ a,b,x $\in$ S ----------- (17)

Replace b by cb in (17), (a(cb) – (cb)a) B(a,x) = 0

Add and subtract (cab) B(a,x) in the last equation

acb B(a,x) – cba B(a,x) + cab B(a,x) – cab B(a,x) = 0

(ac- ca) b B(a,x)+ c(ab – ba) B(a,x) = 0

(ac- ca) b B(a,x) = 0 [by (17)] ----------- (18)

$\therefore$ for each a $\in$ S, either a $\in$ Z(S) or B(a,x) = 0

But Z(S) and Ker B = { a $\in$ S / B(a,x) = 0, $\forall$ x $\in$ S} are additive subgroups of S.

We have either S = Z(S) or S = Ker B

But our assumption, B $\neq$ 0, then S = Z(S).

Therefore S is commutative.

(ii) Linearization on a in (18), we get ((a+d)c – c(a+d)) b B(a+d,x) = 0, $\forall$ a,b,c,d,x $\in$ S

(ac + dc – ca – cd) b [B(a,x) + B(d,x)] = 0

(ac – ca) b B(a,x) + (ac – ca) b B(d,x) + (dc – cd) b B(a,x) + (dc –cd) b B(d,x) = 0

(ac –ca) b B(d,x) + (dc – cd) b B(a,x) = 0

(ac –ca) b B(d,x) = - (dc – cd) b B(a,x)

Pre multiply by (ac – ca) b B(d,x) y on both sides in the last equation we get.

(ac –ca) b B(d,x)y ((ac – ca)b B(d,x)) = - (ac – ca) b B(d,x) y (dc – cd) b B(a,x).

((ac – ca) b B(d,x)) y ((ac – ca) b B(d,x)) = 0 [by (18)]

Since S is semiprime, (ac – ca) b B(d,x) = 0.

In particular, {a B(d,x) – B(d,x) a} b {a B(d,x) – B(d,x) a} = 0

Since S is semiprime, aB(d,x) = B(d,x)a.

ie, B(d,x) $\in$ Z(S), $\forall$ d,x $\in$ S

consequently, B is biderivation.

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